TUTORIAL 1

1. Inverse of a product

Let G be a group and let $a, b \in G$. Prove that

$$(ab)^{-1} = b^{-1}a^{-1}.$$

2. Group isomorphisms

Definition 2.1. *Let $(G, *_G)$ and $(H, *_H)$ be two groups. We say G is* **isomorphic** *to H if there exists a bijection*

$$f : G \to H$$

such that

$$f(g_1 *_G g_2) = f(g_1) *_H f(g_2)$$

for all $g_1, g_2 \in G$. We call f a group isomorphism.

Fix an integer $n > 1$. We have defined groups $\mathbb{Z}_n$ and U_n in class, which looked very similar to each other. Prove that $\mathbb{Z}_n$ is isomorphic to U_n.

3. The inverse of an isomorphism

Let G, H be two groups. Let $f : G \to H$ be an isomorphism. In particular, f is a bijection. Therefore, there is an inverse map $f^{-1} : H \to G$. Prove that f^{-1} is also an isomorphism.

4. Non-isomorphic groups

(1) Prove that $\mathbb{Z}_m$ and $\mathbb{Z}_n$ are not isomorphic when $m \neq n$.
(2) (Difficult) Prove that $(\mathbb{Z}, +)$ and $(\mathbb{Q}, +)$ are not isomorphic.

5. Transposing Matrices

Let A be an $n \times n$ matrix, with entries $[A]_{ij}$ for $1 \leq i, j \leq n$. Recall that A^t is the *transpose* of A, namely the matrix with entries $[A^t]_{ij} = [A]_{ji}$. Prove that

$$(AB)^t = B^t A^t$$

for any other $n \times n$ matrix B.